Does mortality decline always lead to an increase in human capital investment?

Zhipeng Cai [*]

Sau-Him Paul Lau [†]

November 2011

Abstract

According to conventional wisdom in human capital theory, mortality decline leads to more schooling, because individuals expect to have more years to reap the benefits of human capital investment. This result is shown, analytically or computationally, by many researchers. However, the researchers usually represent mortality reductions by changes in survival parameters, with a parameter change causing mortality changes in different ages simultaneously. We use a general non-parametric approach to the survival function, and apply Volterra derivative for a functional to consider the effect of mortality decline at an arbitrary age. This approach enables us to show that mortality reductions at younger versus older ages have systematically different effects on optimal schooling years. In particular, a mortality decline during the studying phase leads to shorter schooling years, providing a counter-example to the conventional wisdom.

JEL Classification Numbers: I20; J10; J24

Keywords: mortality decline; human capital investment; retirement age

[*] School of Economics and Finance, University of Hong Kong, Hong Kong. E-mail: caizp@hku.hk

[†] Corresponding author. School of Economics and Finance, University of Hong Kong, Hong Kong. E-mail: laushp@hku.hk

fi

ff

fi

fi

fi

---

ff
fi

ff

ff

fi

ff

ff

ff

fi

ff

ff

ff fi

fi

ff ff

ff ff

ff

ff

fi

ff
ff
fi
ff
ff

fi

fi

fi

fl
fi

ff

ff

ff

$$l(x) = e^{-\int_0^x \mu(q)dq},$$

$l(x)$ x $l(0) = 1$ $l(T) = 0$ T

$\mu(q) \geq 0$ $\lim_{q \to T} \mu(q) = \infty$

q

S R

---

fl fi

$$a'(x) = [r + \mu(x)]\, a(x) - \phi(x) - c(x)$$

$x \leq S$ $\phi(x)$ x fi

$$h'(S^*) \left[\int_S^R e^{-rx} l(x)\, w(x)\, dx\right] = e^{-rS} \; l(S^*) \left[h(S^*)\, w(S^*) + \phi(S^*)\right].$$

$$\int_0^T \mathrm{e}^{-\rho x} \mathrm{l}(\mathrm{x}) \frac{\mathrm{c}(\mathrm{x})^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \mathrm{dx} - \int_0^R \mathrm{e}^{-\rho x} \mathrm{l}(\mathrm{x}) \, \mathrm{f}(\mathrm{x}) \, \mathrm{dx},$$

$$\mathrm{a}'(\mathrm{x}) = \begin{cases} [\mathrm{r} + \mu(\mathrm{x})] \, \mathrm{a}(\mathrm{x}) + \mathrm{h}(\mathrm{S}) \, \mathrm{w}(\mathrm{x}) - \\ \end{cases}$$

S fi

$\widetilde{R}(S)$

$$e^{-r\widetilde{R}(S)} h(S)\, w\left(\widetilde{R}(S)\right) c\left(0, S, \widetilde{R}(S)\right)^{-\frac{1}{\sigma}} = e^{-\rho \widetilde{R}(S)} f\left(\widetilde{R}(S)\right).$$

fi

$\widetilde{R}(S)$

$$h'(S\ )\ \cdot \int_{S^*}^{R^*} e^{-rx} l(x)\, w(x)\, dx \Big] = e^{-rS^*} l(S\ )\, h(S\ )\, w(S\ ),$$

$$R\ = \widetilde{R}(S\ ),$$

fi

fi S

S R

S

ﬀ x_0

fi

$$\frac{-\partial l(x)}{\partial \mu(x_0)} = \begin{cases} 0 & x < x_0 \\ l(x) & x \geq x_0 \end{cases}.$$

x_0

x_0 ﬀ

R

$l(R)\, f(R)$

$e^{-\ R} l(R)\, f(R)$ fi

$e^{-rR} l(R)\, h(S)\, w(R)$

$c(0, S, R)^{-\frac{1}{}}$

$\frac{-\ l(x)}{C\ (x_0)}$ ﬀ

x x_0 (x)

$\mu(x_0)$ ff [redacted]

$c(0, S, R)$ [redacted]

$\mu(x_0)$ ff $\widetilde{R}(S)$ [redacted]

[redacted]

sign $\frac{\int_{S^*}^{R^*} e^{-rx} \left(\frac{-}{\partial} \right.}{\int_{S^*}^{R^*} e^{-rx} l}$ [redacted] R $\Big($ [redacted]

$\frac{-\partial R}{\partial \mu(x_0)} =$ [redacted]

fi [redacted]

fi

x_0 ff

ff

$l(.)$ [redacted] R

[redacted] $r = \rho$ fi

fi

ff fi fi

ff S

$l(S)$

fi fi

ff

ff fi

∞

$\frac{-\partial \widetilde{R}(S^*,\mu(.))}{\partial\mu(x_0)}$

.

$$\text{sign}\left[\frac{-\partial S}{\partial\mu(x_0)}\right] = \text{sign}\left[\frac{\int_{S^*}^{R^*} e^{-rx}\left(\frac{-\partial l(x)}{\partial\mu(x_0)}\right) w(x)\,dx}{\int_{S^*}^{R^*} e^{-rx} l(x)\,w(x)\,dx} - \frac{\left(\frac{-\partial l(S^*)}{\partial\mu(x_0)}\right)}{l(S)}\right].$$

$$\int_{S^*}^{R^*} e^{-rx}\left(\frac{-\partial l(x)}{\partial\mu(x_0)}\right) w(x)\,dx = \begin{cases} \int_{S^*}^{R^*} e^{-rx} l(x)\,w(x)\,dx & x_0 \le S \\ \int_{x_0}^{R^*} e^{-rx} l(x)\,w(x)\,dx & S < x_0 < R \\ 0 & x_0 \ge R \end{cases},$$

$$\frac{-\partial l(S)}{\partial\mu(x_0)} = \begin{cases} l(S) & x_0 \le S \\ 0 & x_0 > S \end{cases}.$$

$$\text{sign}\left[\frac{\int_{S^*}^{R^*} e^{-rx}\left(\frac{-\partial l(x)}{\partial\mu(x_0)}\right) w(x)\,dx}{\int_{S^*}^{R^*} e^{-rx} l(x)\,w(x)\,dx} - \frac{\left(\frac{-\partial l(S^*)}{\partial\mu(x_0)}\right)}{l(S)}\right] = \begin{cases} 0 & x_0 \le S \\ +1 & S < x_0 < R \\ 0 & x_0 \ge R \end{cases}.$$

$x_0 \leq S$

ff S R

fi

$$\frac{\ldots^{rR^*} l(R^*)w(R^*)}{\int_{S^*}^{R^*} e^{-rx} l(x)w(x)dx}$$

$\frac{-\partial \widetilde{R}(S,\mu(.))}{\partial \mu(x_0)}$

$$\text{sign}\left[\frac{-\partial \widetilde{R}\left(S,\mu\left(.\right)\right)}{\partial \mu\left(x_0\right)}\right] = -\text{sign} \quad \frac{-\partial c\left(0, S, \widetilde{R}\left(S,\mu\left(.\right)\right), \mu\left(.\right)\right)}{\partial \mu\left(x_0\right)} \quad ,$$

$c^{-\frac{1}{\sigma}}$

fi

fi

ff

$$S = S$$

fi

$$\frac{1}{c\left(0, S \;, \widetilde{R}\left(S \;, \mu\left(.\right)\right), \mu\left(.\right)\right)} \quad \frac{-\partial c\left(0, S \;, \widetilde{R}\left(S \;, \mu\left(.\right)\right), \mu\left(.\right)\right)}{\partial \mu\left(x_0\right)}$$

$$= \frac{1}{H\left(S \;, \widetilde{R}\left(S \;, \mu\left(.\right)\right), \mu\left(.\right)\right)} \quad \frac{-\partial H\left(S \;, \widetilde{R}\left(S \;, \mu\left(.\right)\right), \mu\left(.\right)\right)}{\partial \mu\left(x_0\right)}$$

$m\left(0\right)$ $H\left(0, S, R\right)$ $c\left(0, S, R\right)$

$\mu\left(.\right)$

$$+\frac{1}{m(0,\mu(.))}\left(\frac{-\partial m(0,\mu(.))}{\partial\mu(x_0)}\right).$$

$$\frac{1}{m(0,\mu(.))}\left(\frac{-\partial m(0,\mu(.))}{\partial\mu(x_0)}\right)=\frac{-\int_{x_0}^{T}e^{-[(1-\sigma)r+\sigma\rho]x}l(x)\,dx}{\int_0^T e^{-[(1-\sigma)r+\sigma\rho]x}l(x)\,dx},$$

$[0, T]$

T

$$\frac{1}{H\left(S, \widetilde{R}(S, \mu(.)), \mu(.)\right)} \quad \frac{-\partial H\left(S, \widetilde{R}(S, \mu(.)), \mu(.)\right)}{\partial\mu(x_0)}$$

$$= \quad \begin{matrix} 1 \\ \frac{\int_{x_0}^{R^*} e^{-rx}l(x)w(x)dx}{\int_{S^*}^{R^*} e^{-rx}l(x)w(x)dx} \\ 0 \end{matrix} \qquad \begin{matrix} x_0 \leq S \\ S < x_0 < R \\ x_0 \geq R \end{matrix} \quad .$$

$0 < \frac{\int_{x_0}^{R^*} e^{-rx}l(x)w(x)dx}{\int_{S^*}^{R^*} e^{-rx}l(x)w(x)dx} < 1$ x_0 (S, R)

ff

fi

$x_0 < R$

ff ff

$$\text{sign}\left[\frac{-\partial\widetilde{R}(S, \mu(.))}{\partial\mu(x_0)}\right] = \quad \begin{matrix} & -1 & \\ -1 & 0 & +1 \\ & +1 & \end{matrix} \qquad \begin{matrix} x_0 \leq S \\ S < x_0 < R \\ x_0 \geq R \end{matrix} \quad .$$

x_0 $x_0 \geq$
R ff ff

$x_0 \leq S$

ff

S x_0

ff

ff

[illegible]

$x_0 \leq S$

$x_0 \geq R$

$$\text{sign}\left[\frac{-\partial S}{\partial \mu(x_0)}\right] = \text{sign}\left[\frac{\ldots R(S, \mu(.))}{\partial \mu(x_0)}\right] = \begin{cases} -1 & x_0 \leq S \\ 1 & x_0 \geq R \end{cases}$$

ff ff

ff

$S < x_0 < R$

$$\text{sign}\left[\frac{-\partial S}{\partial \mu(x_0)}\right] = \text{sign}\left[\frac{\int_{x_0}^{R^*} e^{-rx} l(x)\, w(x)\, dx}{\int_{S^*}^{R^*} e^{-rx} l(x)\, w(x)\, dx} + \frac{e^{-rR^*} l(R)\, w(R)\left(\frac{-\partial \widetilde{R}(S^*, \mu(.))}{\partial \mu(x_0)}\right)}{\int_{S^*}^{R^*} e^{-rx} l(x)\, w(x)\, dx}\right]$$

$$= -1,\ 0 \quad +1.$$

ff

ff ff

x_0 ff

fi

$\frac{\partial \widetilde{R}(S^*,\mu(.))}{\partial S}\left(\frac{-}{\quad}\right)$ ff

$\frac{(S^*,\mu(.))}{\partial \mu(x_0)}$

$\text{sign}\left[\frac{-\partial \widetilde{R}(S^*,\mu(.))}{\partial \mu(x_0)}\right]$ $\text{sign}\left[\frac{-\partial S^*}{\partial \mu(x_0)}\right]$

sign ... S) / S)] $> 0.$

ff

x_0

$\text{sign}\left[\frac{-\partial}{\partial \mu(x_0)}\right] = \text{sign} \ldots (.) \ldots + \left(\frac{-\partial \widehat{R}\ S\ ,\mu(.}{\partial \mu (x_0)}\right.$

S

$= \ldots 1$ < R .

R

R

ff

fi

ff

ff
ff

ff ff ff

fi ff

fi
ff

S

fi
S

ff

ff ff

ff

[redacted]

fi

fi

0

$$\int_0^T e^{-rx} l(x)\, c(x, S, R)\, dx = \int_S^R e^{-rx} l(x)\, h(S)\, w(x)\, dx,$$

S R

$$\frac{\partial c(0, S, R)}{\partial S} = m(0) \; h'(S) \int_S^R e^{-rx} l(x)\, w(x)\, dx - e^{-rS} l(S)\, h(S)\, w(S) \Big] ,$$

$$\frac{\partial c(0, S, R)}{\partial R} = m(0)\, h(S)\, e^{-rR} l(R)\, w(R)\, .$$

S fi

S

R

$$U(S,R) = \int_0^T e^{-\rho x} l(x) \frac{c(x,S,R)^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} dx - \int_0^R e^{-\rho x} l(x) f(x) dx.$$

R $U(S,R)$ S

R

$$\frac{\partial U(S,R)}{\partial R} = e^{-rR} l(R) h(S) w(R) c(0,S,R)^{-\frac{1}{\sigma}} - e^{-\rho R} l(R) f(R).$$

fi

$l(R)$

$$r - \rho + \frac{f'\left(\widetilde{R}(S)\right)}{f\left(\widetilde{R}(S)\right)} - \frac{w'\left(\widetilde{R}(S)\right)}{w\left(\widetilde{R}(S)\right)} + \frac{e^{-r\widetilde{R}(S)} l\left(\widetilde{R}(S)\right) w\left(\widetilde{R}(S)\right)}{\sigma \int_S^{\widetilde{R}(S)} e^{-rx} l(x) w(x) dx} > 0$$

$\left.\frac{\partial^2 U(S,R)}{\partial R^2}\right|_{R=\widetilde{R}(S)} < 0$

fi

fi

S

$$V(S) = \int_0^T e^{-\rho x} l(x) \frac{c\left(x,S,\widetilde{R}(S)\right)^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} dx - \int_0^{\widetilde{R}(S)} e^{-\rho x} l(x) f(x) dx.$$

S $V(S)$ ff

$$V'(S) = c\left(0,S,\widetilde{R}(S)\right)^{-\frac{1}{\sigma}} \left[h'(S) \int_S^{\widetilde{R}(S)} e^{-rx} l(x) w(x) dx - e^{-rS} l(S) h(S) w(S)\right].$$

fi

$$\frac{w'(S)}{w(S)} + \frac{2h'(S)}{h(S)} - \frac{h''(S)}{h'(S)} - \mu(S) - r - \frac{e^{-r\widetilde{R}(S^*)} l\left(\widetilde{R}(S)\right) w\left(\widetilde{R}(S)\right)}{\int_{S^*}^{\widetilde{R}(S^*)} e^{-rx} l(x) w(x) dx} \frac{\partial \widetilde{R}(S)}{\partial S} > 0$$

fi $V''(S) < 0$

ﬀ $\widetilde{R}(S)$ $\mu(x_0)$

$$\frac{\partial \widetilde{R}(S)}{\partial \mu(x_0)} = \frac{\frac{-1}{\sigma c\left(0,S,\widetilde{R}(S)\right)} \frac{\partial c\left(0,S,\widetilde{R}(S)\right)}{\partial \mu(x_0)}}{r \text{ [redacted] } + \frac{f'\left(\widetilde{R}(S)\right)}{f\left(\widetilde{R}(S)\right)} - \frac{w'\left(\widetilde{R}(S)\right)}{w\left(\widetilde{R}(S)\right)} + \frac{e^{-r\widetilde{R}(S)} l\left(\widetilde{R}(S)\right) w\left(\widetilde{R}(S)\right)}{\sigma \int_S^{\widetilde{R}(S)} e^{-rx} l(x) w(x) dx}}.$$

ﬀ $\widetilde{R}(S)$ S

$$\frac{\partial \widetilde{R}(S)}{\partial S} = \frac{\frac{h'(S)}{h(S)} - \frac{1}{\sigma c\left(0,S,\widetilde{R}(S)\right)} \frac{\partial c\left(0,S,\widetilde{R}(S)\right)}{\partial S}}{r - \rho + \frac{f'\left(\widetilde{R}(S)\right)}{f\left(\widetilde{R}(S)\right)} - \frac{w'\left(\widetilde{R}(S)\right)}{w\left(\widetilde{R}(S)\right)} + \frac{e^{-r\widetilde{R}(S)} l\left(\widetilde{R}(S)\right) w\left(\widetilde{R}(S)\right)}{\sigma \int_S^{\widetilde{R}(S)} e^{-rx} l(x) w(x) dx}}$$

$$\text{sign}\left[\frac{\partial \widetilde{R}(S)}{\partial S}\right] = \text{sign} \quad \frac{h'(S)}{h(S)} - \frac{1}{\sigma c\left(0,S,\widetilde{R}(S)\right)} \frac{\partial c\left(0,S,\widetilde{R}(S)\right)}{\partial S}$$

$$\frac{\partial c\left(0,S\ ,\widetilde{R}(S\ )\right)}{\partial S} = 0.$$

ﬀ

ﬁ

S

(

ff

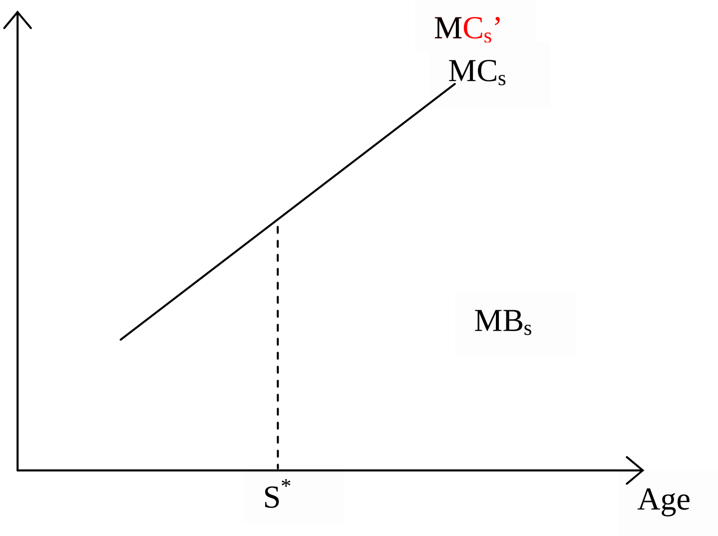
fi fi

ff

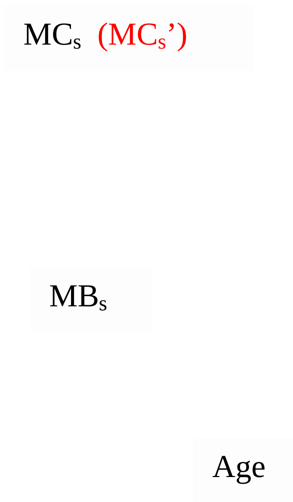
	Survival function	Is retirement age endogenous?	Elements of schooling cost
de la Croix and Licandro (1999)	Exponential	No	Foregone earning
Hu (1999)	Exponential	No	Foregone earning
Bils and Klenow (2000)	Rectangular	No	Foregone earning and tuition cost
Kalemli-Ozcan, Ryder and Weil (2000)	Exponential	No	Foregone earning
Boucekkine, de la Croix and Licandro (2002)	BCL	Yes	Foregone earning
Boucekkine, de la Croix and Licandro (2003)	BCL	No	Foregone earning
Echevarria (2004)	Rectangular	Yes	Foregone earning
Echevarria and Iza (2006)	BCL	Yes	Foregone earning
Hazan (2009)	Rectangular	Yes	Foregone earning
Heijdra and Romp (2009)	Gompertz-Makeham	No	Foregone earning
Cervellati and Sunde (2010)	General	No	Foregone earning

Note: BCL stands for the two-parameter specification used in Boucekkine, de la Croix and Licandro (2002).

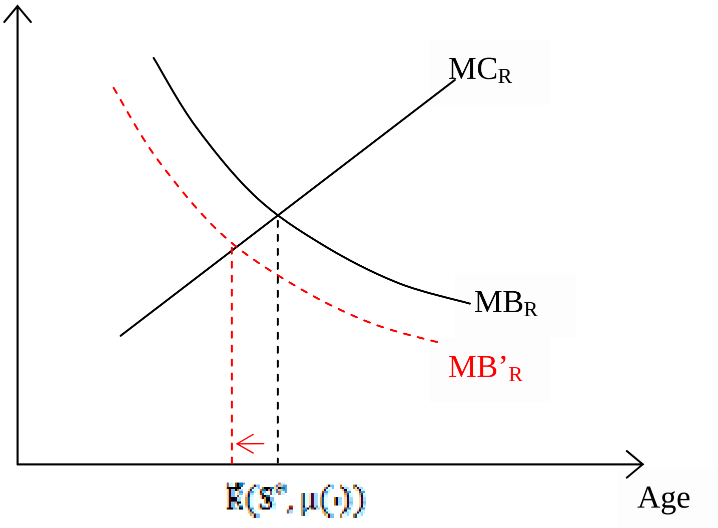
Table 1: Recent Papers on Mortality Decline and Schooling Years



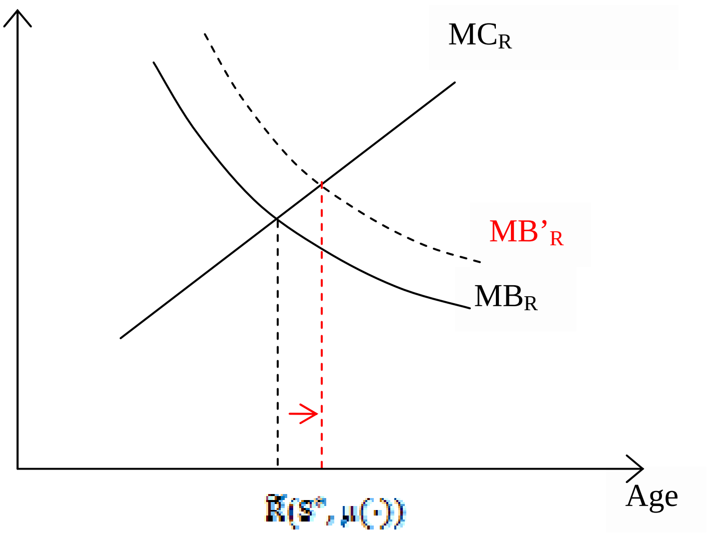
(a) $x_0 \le S^*$



(b) $S^* < x_0 < R^*$



(a) $x_0 \leq S^*$



(b) $x_0 \geq R^*$

Figure 2: Indirect Effects on Schooling Years through Retirement Age